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MATHEMATICS 20 & 23

Alberta
EDUCATION

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MATHEMATICS 20 AND 23
INTERIM CURRICULUM GUIDE

PLEASE NOTE:

This curriculum guide is interim in nature. The guidelines presented in this curriculum guide will be subject to revision according to information received from schools during the school year 1981-82.

A comprehensive curriculum guide and resource guide will be produced when the complete senior high school mathematics program is completed.

The complete program will be implemented as follows:

	OPTIONAL	MANDATORY
Math 10, 13	September, 1980	September, 1981
Math 20, 23	September, 1981	September, 1982
Math 30, 33	September, 1982	September, 1983

NOTE: THIS HANDBOOK IS A SERVICE PUBLICATION ONLY. The information is prescriptive only insofar as the content of the handbook duplicates that of the Senior High Program of Studies.

ACKNOWLEDGEMENT

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Alberta Education also acknowledges the valuable contribution of many senior high school teachers throughout the province in providing assistance in the development of the program.

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ORIENTATION TO THE PROGRAM

A. STRUCTURE

Core

A common set of mathematics objectives has been identified as basic to both the Mathematics 20 and Mathematics 23 course. The topics of this core program are a continuation of those presented in the Grade 10 program.

Trigonometry and Statistics are new to the Grade 10 program. These topics are further developed as complete strands through the remaining two high school grades.

It should be noted that even though some of the objectives are similar for the two courses the methods of instruction and the time required for topics will not be the same. An experimental approach with more time being devoted to basic skills is suggested for the Mathematics 23 program.

Independent Core

While common objectives of topics have been identified as core, each course has a separate set of objectives specific to the course. These objectives are listed under the title of independent core.

The mathematics 20 independent core goes further into the theoretical development of mathematics concepts with a greater emphasis on formalization.

The model of instruction suggested for the Mathematics 23 independent core would be based on an experimental approach with an increase in time spent on exploratory activities.

Electives

Elective topics in the senior high school mathematics program have been included only at the Grade 11 and 12 levels. The grade 10 program does not have an elective component to ensure sufficient time for an adequate mathematical foundation for subsequent courses.

B. AREAS OF EMPHASIS

Course Content

A deliberate attempt has been made to reduce course content with possible gains in the form of learning by discovery. The number of concepts and the depth of coverage has been reduced in an attempt to provide time for teachers and students to pursue different interesting ideas.

Problem Solving

A deliberate attempt has been made to stress problem solving. This is a crucial aspect of mathematical activity and as such should demand a great deal of effort, activity and planning on the part of teachers and students.

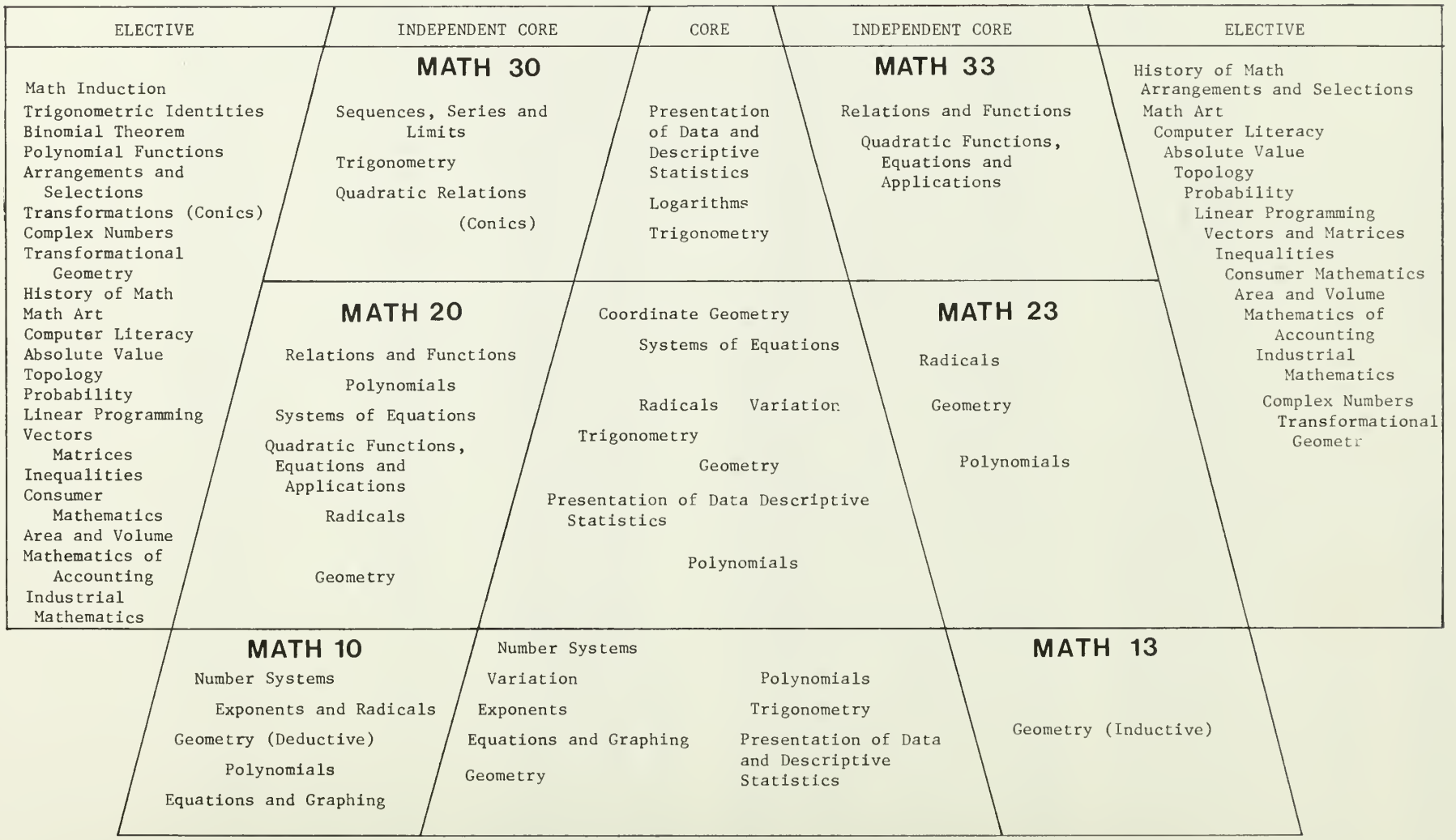
Application

A deliberate attempt has been made to relate mathematics to real life situations with a view to making the study of mathematics more meaningful to the vast majority of students.

Resource Material

A section of comments and possible applications has been prepared to give some suggestions to teachers. This section will be greatly improved as more ideas are supplied by teachers. A comprehensive resource guide including references, materials, films and other aids will be produced as the complete high school program is developed.

MATHEMATICS PROGRAM OVERVIEW



SUGGESTED TIME ALLOCATIONS

Math 20

	Topic	Number of Hours
A.	Radicals	7
B.	Polynomials	6
C.	Co-ordinate Geometry	15
D.	Presentation of Data and Descriptive Statistics	6
E.	Relations and Functions	12
F.	Quadratic Functions, Equations and Applications	18
G.	Systems of Equations	12
H.	Geometry	15
I.	Trigonometry	10
J.	Variation	6
K.	Electives	18
		125

Math 23

	Topic	Number of Hours
A.	Radicals	12
B.	Polynomials	12
C.	Co-ordinate Geometry	20
D.	Presentation of Data and Descriptive Statistics	10
E.	Relations and Functions (Math 20 only)	-
F.	Quadratic Functions, Equations and Applications (Math 20 only)	-
G.	Systems of Equations	16
H.	Geometry	15
I.	Trigonometry	15
J.	Variation	7
K.	Electives	18
		125

TEXTS AND REFERENCES

MATHEMATICS 20:

Prescribed Reference: MATH IS/5
Frank Ebos, Bob Tuck
Nelson/Canada Ltd., 1980

Prescribed Reference: HOLT MATHEMATICS 5
Kenneth D. Fryer, Ronald G. Dunkley,
H.A. Elliott, Norman J. Hill, R. Jock MacKay
Holt, Rinehart and Winston of Canada Ltd., 1980

Recommended Reference: FOUNDATIONS OF MATHEMATICS FOR TOMORROW:
INTERMEDIATE
Dino Dottori, George Knill, James Stewart
McGraw-Hill Ryerson Ltd., 1978

MATHEMATICS 23:

Prescribed Reference: APPLIED MATHEMATICS FOR TODAY: INTERMEDIATE
Book 1 Second Edition
Dino Dottori, George Knill, John Seymour
McGraw-Hill Ryerson Ltd., 1976

Recommended Reference: MATHEMATICS FOR A MODERN WORLD
Book 3 Second Edition
A.G. Burns, R.G. Pinkney, J.J. Del Grande
Gage Publishing Ltd., 1976

GOALS OF THE SENIOR HIGH SCHOOL MATHEMATICS PROGRAM

Although the different courses of the senior high school mathematics program have different specific objectives, the goals of the senior high mathematics program are set forth in relation to three main expectations and needs: those of the individual, those of the discipline of mathematics and those of society at large. They are listed as follows:

Student Development

a) To develop in each student a positive attitude towards mathematics.

b) To develop an appreciation of the contribution of mathematics to the progress of civilization.

c) To develop the ability to utilize mathematical concepts, skills and processes.

d) To develop the powers of logical analysis and inquiry.

e) To develop an ability to communicate mathematical ideas clearly and correctly to others.

Discipline of Mathematics

a) To provide an understanding that mathematics is a language using carefully defined terms and concise symbolic representations.

b) To provide an understanding of the concepts, skills and processes of mathematics.

c) To provide an understanding of the common unifying structure in mathematics.

d) To furnish a mode of reasoning and problem solving with a capability of using mathematics and mathematical reasoning in practical situations.

Societal Needs

a) To develop a mathematical competence in students in order to function as citizens in today's society.

b) To develop an appreciation of the importance and relevance of mathematics as part of the cultural heritage that assists people to utilize relationships that influence their environment.

c) To develop an apprecation of the role of mathematics in man's total environment.

PROBLEM SOLVING

The discipline of Mathematics provides one of the best opportunities to practise the process of problem solving. The techniques learned and experience gained are transferable to almost any problem solving situation and provide the basis of successful living.

Everyone's job involves problem solving. Although many of the problems we have to contend with are repetitious and are easily resolved once the initial solution is known, the most challenging and exciting aspects deal with problems that are completely new to the individual or arriving at solutions in new and creative ways.

A primary purpose of education is to provide a student with the tools to solve a wide variety of problems. Specific problem solving skills can be taught; however, the methods of successfully tackling untried problems is not always fully understood. There is a sequence of steps by which one can proceed, such as:

a) fully understanding the problem

b) accepting the challenge of searching for a solution

c) making a reasonable guess at the answer (this provides a check when a solution is finally found)

d) pictorially or graphically respresenting the important aspects of the problem if this is at all possible

e) solving a simpler version of the problem or a special case of the more general problem

f) using specific cases to discover patterns or different approaches to the problem

g) solving the general problem

h) checking the solution in the special cases solved before (Does the solution seem reasonable based on your initial guess?)

The above sequence of steps is helpful in tackling any problem. Considerable intuition and experience are still necessary to tackle the really difficult problems.

APPLICATIONS

A central and encompassing approach to making the study of mathematics both meaningul and interesting is to place a major emphasis in the area of applications. An application is the process of using a mathematical skill to arrive at a solution to a real life or practical situation. Applications should incorporate interesting, useful, relevant and diverse examples from real life situations. The significance of using applications is based on relating mathematical conceps and skills to problems encountered in society and the environment. Mathematics may be related to countless aspects of living. Our task is to have students recognize these relations, develop an understanding of the interrelationships that exist with a mathematics program and then learn to transfer their present use of mathematics to other situations.

Applications, like problem solving, should be integrated into the overall program rather than dealt with as an independent unit. Whenever possible, integration and coordination with other subject areas is important. When applications require extensive computations, the use of the calculator may become a necessary component of the learning process to avoid time-consuming calculations.

STATISTICS

Statistics is an exciting new addition to the mathematics curriculum. It provides many opportunities for applications and hence is easily motivating.

Statistics, over the past number of years, has played an increasing role in public information. We are continually bombarded by statistical information, such as Consumer Price Index, baseball averages, weather forecasting, election polls and stock market indices.

Statistics is being used extensively in research and industry. In any enterprise where large amounts of data need to be handled and processed, where predictions are to be made, statistical methods play a role. Statistical techniques are also used in designing telephone exchanges, optimally setting traffic lights, designing insurance policies, determining the reliability of one's car, criminal detection, and market research, to mention a few.

The introduction of statistics at this level is intended to familiarize the student with the elementary descriptive measures that form the basis of any further work with large data sets. In most cases the data should be collected by the student, preferably different data for each student. This could be a stimulating project. The subsequent analysis then becomes more meaningful and hopefully provides not only statistical insight, but some new knowledge about our surroundings. In order to understand and intelligently discuss much of the information to which we are daily subjected, some knowledge of the terminology and underlying assumptions of statistics is necessary.

Statistics is an applied science. To motivate this subject, especially at the introductory level, one should concentrate on the experimental approach. This unit provides the opportunity for field trips to collect data. By displaying this data in various forms via graphs, one can make some basic inferences about the data source. To justify the collection of the data, it is important to make some inference. Just as important is to ask what further information would be worth knowing and discuss how one would proceed to discover it.

THE ELECTIVE COMPONENT

The elective component is an integral part of the senior high school program at the Grade 11 and 12 levels. The elective component primarily offers an opportunity for students to spend time on interesting and useful areas of mathematics not necessarily contained in the core and independent core component.

The time allotment for the elective component should be approximately 18 hours at the Grade 11 level. It is suggested that the 18 hours of electives be determined according to the plans of the individual teacher.

A. Structure of the Elective

The elective component of the program may be:

1) a content area not prescribed as a core or independent core topic

OR

2) a locally developed unit as determined by the teacher or school system

OR

3) an extension of the subject matter in any of the core or independent core topics to provide students with enrichment.

B. Guidelines

Teachers should keep the following guidelines in mind:

1) The topics are open-ended so that the interests and abilities of students may be taken into account.

2) Student initiated projects may be considered as an elective.

3) A teacher should make use of any appropriate resources.

4) Electives should be included in the course throughout the year, wherever appropriate. These should not necessarily be taught during an 18 hour block. Where more than one elective is included, the time for each elective does not necessarily have to be the same.

5) The elective component of the program should be included in the evaluation of the students.

6) Prerequisite core material may be required before some electives are attempted.

C. Suggested Time Allotments

The time allocated to topics in the elective component of the course may vary according to the topic(s) chosen and instructional preferences of the teacher. A total of 18 hours may be devoted to cover one elective topic, or alternatively, two or more topics may be covered. The time allotted to the elective topic is the discretion of the teacher. Total time for electives is 18 hours.

Example 1: Computer Literacy covering all 18 hours.

Example 2: Consumer Mathematics/Trigonometric Identities covering 18 hours.

Example 3: Vectors/Math Art/Inequalities covering 18 hours.

D. Suggested Elective Outlines

The elective component is designed to be an interesting and motivational aspect of the mathematics program. The outlines (on pages 54 to 89) for elective topics are intended to act as guidelines for individual teachers. You may wish to follow the suggested outlines or incorporate your own ideas for the elective component. The primary references provide useful material for many of the elective nits.

E. Provision for the Academically Talented

Topics within the elective component of the course should be utilized to challenge the academically talented students. Such topics as computer literacy and topology, for example, may be used to provide challenge to students of this category. Teachers may also extend core topics and concepts to a higher level of complexity to meet the needs of the stronger academic student.

MATHEMATICS 20 AND 23
COURSE OBJECTIVES

A. RADICALS

MATH 20	MATH 23	COMMON CORE	
	▒		1. Review the basic laws of exponents.
	▒		2. Identify each part of a radical expression.
▒			3. Maintain previous skills.
		▒	4. Simplify radical expressions.
		▒	5. Perform the four basic operations on radicals of the form $\sqrt[b]{x}$, b = 2, 3.
		▒	6. Rationalize radical denominators that are monomials and binomials.
		▒	7. Solve radical equations containing one radical in one variable.
▒			8. Solve radical equations containing two radicals in one variable.

B. POLYNOMIALS

		▒	1. Maintain previous skills in algebraic operations.
		▒	2. Maintain previous skills of factoring.
		▒	3. Maintain previous skills of solving linear equations with one unknown.
▒			4. Factor polynomials of the form $p^3 \pm q^3$.
▒			5. Factor polynomials which are incomplete squares.

MATH 20	MATH 23	COMMON CORE	
			6. Factor polynomials by the grouping method.
			C. COORDINATE GEOMETRY
			1. Maintain previous skills related to the following: quadrants, axis, origin and ordered pairs.
			2. Determine the distance between two points.
			3. Determine the coordinates of the midpoint of a line segment.
			4. Determine the slope of a line passing through two given points.
			5. State the relationship between slopes of: a. parallel lines b. perpendicular lines
			6. Graph lines whose equations are in the form $Ax + By + C = 0$ or $y = mx + b$ by: a. using ordered pairs b. using the intercepts c. using the slope and y-intercept
			7. Interpret graphs of straight lines.
			8. Use the slope test to determine whether three points are collinear.
			9. State the intercepts of a line by examining the graph.
			10. Determine the intercepts of a line from its equation.
			11. Write an equation and draw the graph of: a. a vertical line b. a horizontal line

MATH 20	MATH 23	COMMON CORE	
			12. Write an equation of a line and draw its graph given the slope and a point on the line.
			13. Write the equation of a line passing through a given point and a. parallel to a given line b. perpendicular to a given line
			14. Given two points, a. draw the graph of the line passing through them b. write the equation of the line passing through them
			D. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS
			1. Maintain previous skills.
			2. Draw histograms and cumulative frequency histograms (ogives).
			3. Demonstrate how to obtain medians, quartiles and percentiles (graphically and from grouped data).
			4. Determine the dispersion of a variable by: a. using percentiles b. calculating the standard deviation
			E. RELATIONS AND FUNCTIONS
			1. Define a relation.
			2. Define the domain and range of a relation and its inverse.

MATH 20	MATH 23	COMMON CORE	
			3. Determine the inverse of a relation.
			4. State the relationship between the domain and range of a relation and the domain and range of its inverse.
			5. State the relationship between the graph of a relation and the graph of its inverse.
			6. Define a function.
			7. Use the functional notation f(x) in defining a function.
			8. For particular values of x, find f(x).
			9. Define and graph a linear function.
			F. QUADRATIC FUNCTIONS, EQUATIONS & APPLICATIONS
			1. Identify and express quadratic functions in the form $y = ax^2 + bx + c$ where $a, b, c \in R, a \neq 0$.
			2. Identify and express quadratic equations in the form $ax^2 + bx + c = 0$ where $a, b, c \in R, a \neq 0$.
			3. Graph a quadratic function using a table of values.
			4. Find the vertex, axis of symmetry, domain, range and maximum or minimum value of a quadratic function from its graph.
			5. Use the formula for vertex and axis of symmetry if the quadratic function is given in the form $y = ax^2 + bx + c$.

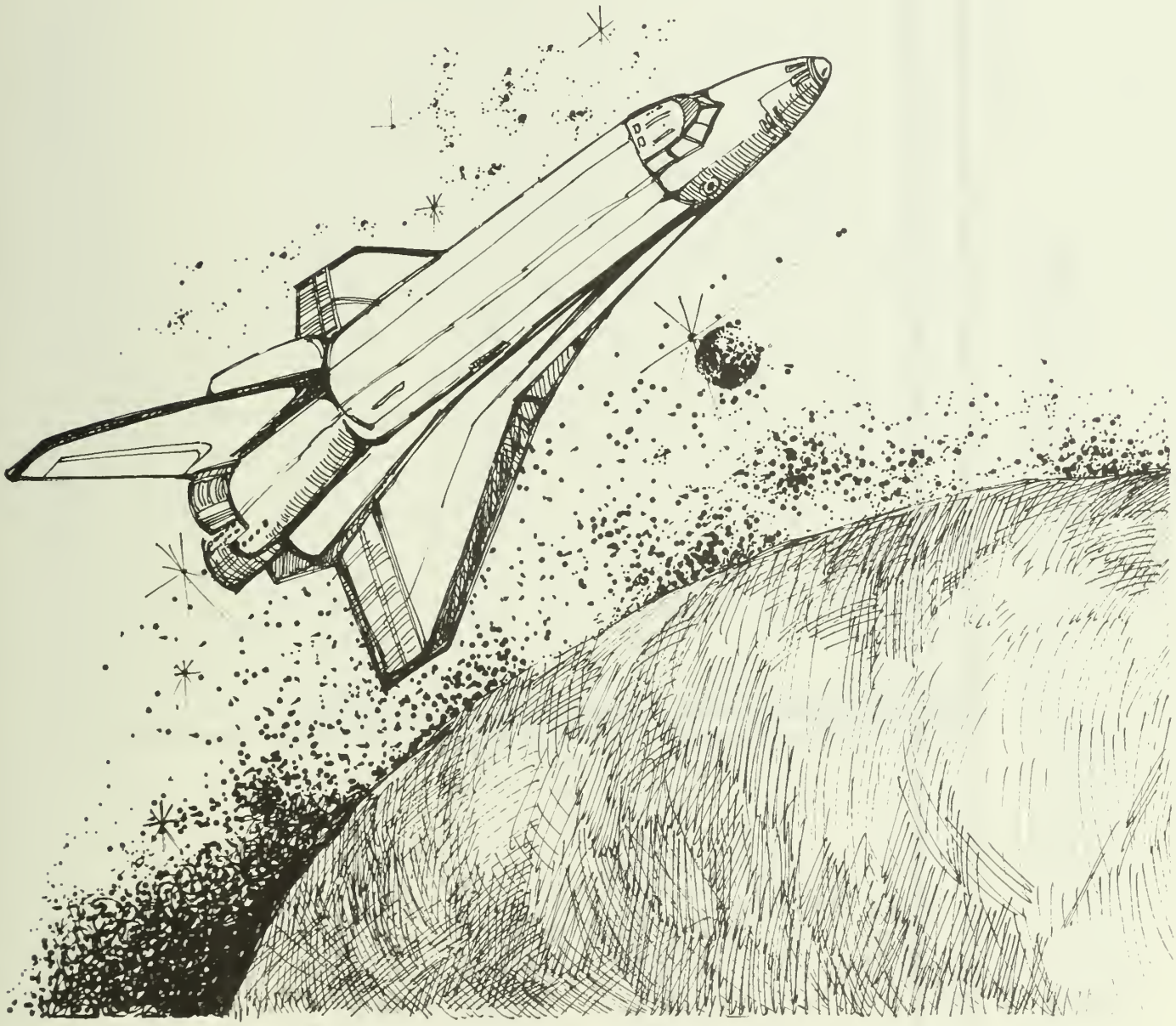
MATH 20	MATH 23	COMMON CORE	
			6. State the relationship between the graph of a quadratic function and the roots of the corresponding equation.
			7. Write quadratic equations in the form $ax^2 + bx + c = 0$ and specify the value of a, b, c.
			8. Use the method of completing the square of a quadratic function to find the vertex, axis of symmetry, range and maximum or minimum value. Draw the graph using this information.
			9. Solve problems involving the maximum or minimum value of a quadratic function.
			10. Compute the real roots of a quadratic equation by: a. factoring b. using the quadratic formula c. completing the square
			11. Define and evaluate the discriminant of a quadratic equation.
			12. State the nature of the roots by examining the discriminant.
			13. Solve problems whose solutions are based on quadratic equations.
			G. SYSTEMS OF EQUATIONS
			1. Solve linear systems of equations both algebraically and graphically.
			2. Identify systems as having many solutions, one solution or no solution.
			3. Solve problems based on the solution of systems of equations.

MATH 20	MATH 23	COMMON CORE	
▒			4. Solve linear-quadratic and two-quadratic systems.
			H. GEOMETRY
		▒	1. Define and illustrate the following terms related to the circle: radius, chord, interior, exterior, arc, semi-circle, segment, sector, central angle, inscribed angle, secant line and tangent line.
	▒		2. Discover by experimentation the following circle relationships: a. A line through the centre of a circle and the midpoint of a chord is perpendicular to the chord. (Converses should also be considered). b. The measure of an inscribed angle is half the measure of the central angle subtended by the same arc (or congruent arcs). c. Inscribed angles subtended by the same arc (or congruent arcs) are congruent. d. An angle inscribed in a semi-circle is a right angle. e. A tangent line is perpendicular to the radius drawn to the point of contact. f. Tangent segments drawn to a circle from the same exterior point are congruent.
	▒		3. Solve problems involving numerical applications of the relationships in objective 2.
▒			4. Apply the following basic theorems: a. A line containing the centre of a circle that bisects a chord, which is not a diameter, is perpendicular to that chord. (Include corollaries) b. A tangent line is perpendicular to the radius drawn to the point of contact.

MATH 20	MATH 23	COMMON CORE	
			c. The measure of an inscribed angle is half the measure of the central angle subtended by the same arc (or congruent arcs).
			5. Construct a tangent to a circle from a given point.
			6. Find the length of an arc given the measures of the central angle and the radius.
			7. Find the area of a sector given the measures of the central angle and the radius.
			8. Solve problems involving numerical applications of the above theorems.
			I. TRIGONOMETRY
			1. Maintain previously developed skills in solving right triangles.
			2. Define secant, cosecant and cotangent ratios.
			3. Determine the secant, cosecant and cotangent values of acute angles.
			4. Determine the measure of any acute angle given the secant, cosecant and cotangent ratio.
			5. Solve right triangles using trigonometric ratios.
			6. Solve problems involving right triangles including angles of elevation and depression.

MATH 20	MATH 23	COMMON CORE	
			7. Determine the relative measures of the sides of: a. a 30 - 60 - 90 triangle b. a 45 - 45 - 90 triangle
			J. VARIATION
			1. Maintain previous skills of direct, inverse and partial variation.
			2. Identify joint variation.
			3. Solve problems related to joint variation.

MATHEMATICS 20:
PRIMARY REFERENCE CORRELATION



MATHEMATICS 20:
PRIMARY REFERENCE CORRELATION

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	PRESCRIBED REFERENCES MATH IS/5
A. RADICALS				
1. 2.	NOTE: Objectives 1 and 2 are taught only in Math 23.			
3. Maintain previous skills.	Review of concepts from Math 10; refer to comments on "Exponents and Radicals" in Math 10.			92-94
4. Simplify radical expressions.	For radicals of the form $\sqrt[b]{x^m y^n}$ where b = 2 or 3 and m and n are positive integers.		35 - 38	95,96
5. Perform the four basic operations on radicals of the form $\sqrt[b]{x}$, b = 2,3	Same type of radicals as above; addition and subtraction can be explained by using the distributive law for algebraic expressions: $2x + 3x = (2 + 3)x = 5x$ for radicals $2\sqrt{5} + 3\sqrt{5} = (2+3)\sqrt{5} = 5\sqrt{5}$		36 - 40,46	96-104

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	PRESCRIBED REFERENCES MATH IS/5
6. Rationalize radical denominators that are monomials and binomials.	Demonstrate the reason for rationalizing. It is easier to write $\frac{\sqrt{10}}{2}$ as a decimal than the unrationalized form of $\frac{\sqrt{5}}{\sqrt{2}}$ which requires division of an approximate decimal.		37,40, 46	104-107
7. Solve radical equations containing one radical in one variable.	Emphasize isolation of radical on one side of the equation before squaring.	Radical equations can be applied to determine: a) the braking distance of a car, knowing its speed and type of road. Use: $s = \sqrt{30fd}$, where s = speed, f = coefficient of friction and, d = braking distance (skid marks). The formula can also be used to find speed by measuring the skid marks and thus determining whether the driver was speeding. b) the duration of a hurricane or tornado. Use: $t = 2\frac{d^3}{216}$ or $t = \frac{2d\sqrt{d}}{6\sqrt{6}} = 2d\frac{\sqrt{6d}}{36} = \frac{d\sqrt{6d}}{18}$ in simplified form, where t = time in hours and d = diameter of storm in km.	41-43,46	108-111

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	PRESCRIBED REFERENCES MATH IS/5
8. Solve radical equations containing two radicals in one variable.	Before squaring, ensure that one radical appears on each side of the equation.		41-43,46	109-111
B. POLYNOMIALS				
1. Maintain previous skills in algebraic operations.	This includes working with polynomials and rational expressions.	How many games are necessary if each team in an "n" team lea ue is to play each other twice, once at home and once away? Suppose there are 90 games, how many teams are there?	1-6,18-21,	42-45
2. Maintain previous skills of factoring.	It may be useful to use a flow chart in this type of factoring.		7 - 12	47-54
3. Maintain previous skills of solving linear equations with one unknown.				45,46 55
4. Factor polynomials of the form $p^3 \pm q^3$.	This should include grouping to obtain a common factor and grouping to difference of squares.	Polynomials arise from many problem situations. For example: a) Distance objects fall during a given time. b) The lead distance necessary for a runner on the outside lane of a racetrack. (Math Is 3, 4). c) Problems involving simplifying calculations.	17,18	65,67,68
5. Factor polynomials which are incomplete squares.			22, 23	55
6. Factor polynomials by the grouping method.			7,8,9,10	47,48,53

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	MATH IS/5
C. COORDINATE GEOMETRY				
1. Maintain previous skills related to the following: quadrants, axes, origin and ordered pairs.	The usefulness of mathematics increased greatly when mathematics applied algebraic methods to geometry (in the 17th century) to form a subject called coordinate geometry. Most students are familiar with some aspects of coordinate geometry because they probably have done some work with graphing. (Holt Math 4, p. 115-118) (Math Is/4, p. 168)	Telephone long distance rates are determined by a rectangular grid system. Contact your local telephone business office.		
2. Determine the distance between two points.	Using two points on a graph, say P(1,2) and Q(5,4) derive the distance formula using the Pythagorean Theorem (a review item) before generalizing the formula.	Determine length of rafters, support cables, etc.	103	228-233
3. Determine the coordinates of the midpoint of a line segment.	By taking two points on a graph, the students can predict the midpoint before deriving the midpoint formula.		104-106	214,215
4. Determine the slope of a line passing through two given points.	Since coordinate geometry is highly visual, students should be encouraged to do neat work so that they could better predict the slope of a line, state relationships between slopes of lines, graph lines from $Ax + By + C = 0$ or $y = mx + b$, and interpret graphs of straight lines.		63-66	197-200

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	MATH IS/5
5. State the relationship between slopes of: a) parallel lines b) perpendicular lines.			71-73 73-75	204-207
6. Graph lines whose equations are in the form $Ax + By + C = 0$ or $y = mx + b$ by: a) using ordered pairs b) using the intercepts c) using the slope and y-intercept			61-66	121-123 195 201-203
7. Interpret graphs of straight lines.				
8. Use the slope test to determine whether three points are collinear.	Example: Determine if P(-4,1), Q(2,3) and R(11,6) are collinear. Slope Test: $m_{PQ} = \frac{1 - 3}{-4 - 2} = \frac{-2}{-6} = \frac{1}{3}$ $m_{QR} = \frac{3 - 6}{2 - 11} = \frac{-3}{-9} = \frac{1}{3}$		65,66	198-200
9. State the intercepts of a line by examining the graph.	To find the x-intercept, the student must understand that along the x-axis, y=0. By putting y=0 in the equation one can get the x-intercept. Similarly, by putting x=0 one can get the y-intercept.		61,62,63	
10. Determine the intercepts of a line from its equation.			69-71	194-196

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	MATH IS/5
11. Write an equation and draw the graph of: a) a vertical line b) a horizontal line.			62	
12. Write an equation of a line and draw its graph given the slope and a point on the line.	Again, students should be encouraged to draw reasonable sketches if they are to obtain the required equation or graph. (Math Is/4, p. 199-201)	NOTE: Many good examples of problems related to this section are found in various books. The reference to be noted is the recent NCTM book, A Sourcebook of Applications of School Mathematics.	68	
13. Write the equation of a line passing through a given point and a) parallel to a given line b) perpendicular to a given line.			71-73, 73-75	214-216
14. Given two points: a) draw the graph of the line passing through them b) write the equation of the line passing through them.	(Math Is/4, p. 199-201).		67,68,69	
D. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS				
1. Maintain previous skills.			273-275	264-267 273-277

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	MATH IS/5
2. Draw histograms and cumulative frequency histograms (ogives).	Draw histograms as a review of previous material. (See suggestions in Math 10 applications). Use these histograms to now construct a cumulative frequency histogram. Graphically, (see page 383 of Math Is/4) one can now determine any desired percentile.		278-280, 286	270-272
3. Demonstrate how to obtain medians, quartiles and percentiles (graphically and from grouped data).			285-287	
4. Determine the dispersion of a variable by: a) using percentiles b) calculating the standard deviation.	The dispersion or variability of data is important in any further inference on the source of the data. The standard deviation is the most commonly used measure of dispersion. In many cases, approximately 65% of all data values lie between one standard deviation on either side of the mean. In further statistical work, most tests of inference require a knowledge of the standard deviation. In data with outliers, namely data with a number of small and/or large values, the standard deviation does not accurately represent the dispersion of most of the data. In this case one could perhaps only consider data between the 10th and 90th percentile and use these percentiles as a dispersion measure.	For example, if one wishes to purchase a rifle, there is a considerable range of prices. In this example one pays the most for a rifle with the minimum dispersion, namely the most accurate gun. The dispersion of a rifle can be measured by locking it in a vice and firing a number (say 10) shots into a target. The more scattered the shots appear on the target, the greater the dispersion. As a class project, one could have each student throw ten darts into a dart board. Measure the distances from the bullseye for each dart thrown by a particular student and calculate the standard deviation of these numbers. The student with the smallest standard deviation is the best dart player. Another example is the game of roulette. Consider two players, Art and Brunhilda; Art always bets a dollar on either red or black, while Brunhilda always bets a		277-281

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	MATH IS/5
		dollar on one of the numbers. After playing all evening, Art and Brunhilda can expect to have lost the same amount of money. However, during the roulette game, Art wins approximately half the time, while Brunhilda wins on the average once in every 38 spins of the wheel. Brunhilda's fortune fluctuates wildly while Art's has minimal variability. Since the mean winnings are the same, one can analyze the psychology of a bettor on the basis of the variability of the option he plays.		
E. RELATIONS AND FUNCTIONS				
1. Define a relation.	Relations may be defined by ordered pairs, graphs or open sentences. Family relations actually are examples which can be used to illustrate mathematical relations.	According to a Texaco advertisement, it costs $200,000 to drill one kilometre deep, $450,000 to drill for two kilometres and $1,000,000 to drill for three kilometres. a) Graph these points, sketch a curve through them (not a line, for sure) and estimate the cost of drilling 2.4 kilometres. b) Extend the curve to cross the vertical axis and this estimates the "fixed costs" of drilling in this situation - the cost even if no hole is drilled.	49	190,441
2. Define the domain and range of a relation.			52,53	190
3. Determine the inverse of a relation.			54-57	445,446

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
4. State the relationship between the domain and range of a relation and the domain and range of its inverse.			54-57	
5. State the relationship between the graph of a relation and the graph of its inverse.			55	446
6. Define a function.	A function may be defined as "a relation which has no 'fickle pickers'". The following examples will illustrate this: $R = \{(2,3), (2,4), (3,5),\}$ may be represented as: $2 \rightarrow 3$ Using the matching notation $\searrow 4$ 2 is a "fickle picker" because $3 \rightarrow 5$ it picks more than one member of the range. $S = \{(2,3), (4,5), (6,7)\}$ may be represented as: $2 \rightarrow 3$ There are no "fickle pickers" $4 \rightarrow 5$ so this relation is a function. $6 \rightarrow 7$		50	191,441
7. Use the functional notation f(x) in defining a function.			50-52	37
8. For particular values of x, find f(x).			50,52	37,38 193

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES HOLT MATH 5	PRESCRIBED REFERENCES MATH IS/5
9. Define and graph a linear function.			61-63	194-196
F. QUADRATIC FUNCTIONS, EQUATIONS AND APPLICATIONS				
1. Identify and express quadratic functions in the form $y = ax^2 + bx + c$ where $a, b, c \in R, a \neq 0$.			295	412
2. Identify and express quadratic equations in the form $ax^2 + bx + c = 0$ where $a, b, c \in R, a \neq 0$.			295	427
3. Graph a quadratic function using a table of values.	Students should become familiar with the graphs of quadratic functions of the forms: $y=ax^2$, $y = ax^2 + bx$, and $y = ax^2 + bx + c$.		295-296	413-417
4. Find the vertex, axis of symmetry, domain, range and maximum or minimum value of a quadratic function from its graph.	The importance of the vertex and axis of symmetry can be illustrated using appropriate problem solving applications. It is suggested that the formula be derived before it is used.	A road passes under a railroad overpass in the form of a parabolic arch 5 metres high (at the apex) and 20 metres wide with equation $y = 5 - \frac{x^2}{20}$. Find the height of the tallest truck that can pass under the arch. Assume the truck is 3 metres wide, that it stays on its own side of the road, and that the centerline of the road passes directly under the apex of the arch.	295-302	421-423

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
5. Use the formula for vertex and axis of symmetry if the quadratic function is given in the form $y = ax^2 + bx + c$.			301	421,422
6. State the relationship between the graph of a quadratic function and the roots of the corresponding equation.			304-306	427
7. Write quadratic equations in the form $ax^2 + bx + c = 0$ and specify the value of a, b, c.	Initial graphing of quadratics by a table of values acquaints students with the more important points of a quadratic and also develops an appreciation for graphing quadratics using these critical points.		308-310	431
8. Use the method of completing the square of a quadratic function to find the vertex, axis of symmetry, range and maximum or minimum value. Draw the graph using this information.			300-302	418-421
9. Solve problems involving the maximum or minimum value of a quadratic function.		a) The Wishbones have 30 metres of fence with which to make a rectangular dog run. If they use a side of the house as one side of the run, what dimensions will give the maximum area?	302,303	424-427

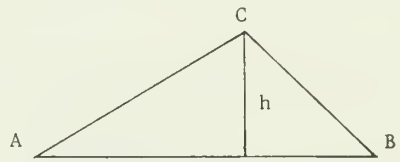
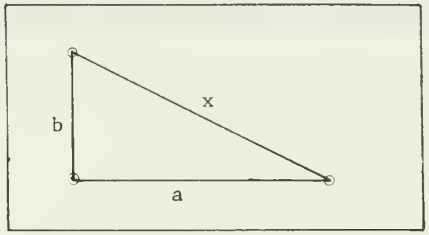
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
		b) Fast Freddie's Theatre charges $4.00 per ticket, and it has had a full house of 400 nightly. The manager estimates that the ticket sales would decrease by 50 for each $1.00 increase in the ticket cost. What is the most profitable price to charge? (A simple arithmetic solution using a table of values could be used to introduce the concept of maximum value). eg.: N | 400 | 350 | 300 | 250 U | 4 | 5 | 6 | 7 I | 1600 | 1750 | 1800 | 1750 Max c) The collision impact (I) of an automobile with mass (m) and speed (s) is given by the formula $I = kms^2$. If the speed is tripled, what happens to the collision impact of a 1000 kg car?		
10. Compute the real roots of a quadratic equation by: a) factoring b) using the quadratic formula c) completing the square.			304-310	55-58 430,431 427-429
11. Define and evaluate the discriminant of a quadratic equation.			309,310	435

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
12. State the nature of the roots by examining the discriminant.			309,310	435,436
13. Solve problems whose solutions are based on quadratic equations.		a) A club bought a snowmobile for $720.00, planning to divide the expenses equally among the members. However, two members withdrew from the club, increasing each share by $4.00. How many members were in the club originally? b) How wide a strip must be sodded around a square court 10 metres on a side, so that one-half of the court is sodded? c) A 6 centimetre square is cut from each corner of a square piece of sheet metal. The sides are folded up to form an open box having a volume of 600 cc. What is the length of a side of the original square piece of sheet metal? d) Two students agree to share the work of cutting the grass on a large lot measuring 80 metres by 60 metres. One student starts by cutting a strip all of the way around the lot and continuing around in this way. How wide will this strip be when half the work is completed?	310-314	431-434

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
G. SYSTEMS OF EQUATIONS				
1. Solve linear systems of equations both algebraically and graphically.	It would be worth introducing solution by graphing before the algebraic solutions are taught. In the graphical solution a student has a chance to understand how equivalent systems can be generated and used to arrive at a solution. More sophisticated methods such as general formulas and determinants may be left to better students for further exploration. In actual practice, a combination of addition-subtraction and substitution will probably provide the best method of solution.	Problems involving two vehicles travelling at different speeds and leaving a fixed point at different times can be used. Using graphical or algebraic means, we can find out when the two vehicles meet.	85-87	120-126 128-138
2. Identify systems as having many solutions, one solution or no solution.	Systems can be described graphically showing: a) coincident lines (dependent system) b) parallel lines (inconsistent) c) intersecting lines (independent). Students should be able to determine the type of system by examining the slopes of the lines.		83-85	127
3. Solve problems based on the solution of systems of equations.	Problems involving statements about numbers, supplementary and complementary angles, speeds of two vehicles, digits of numbers and investments can be used.		93-95	139-155

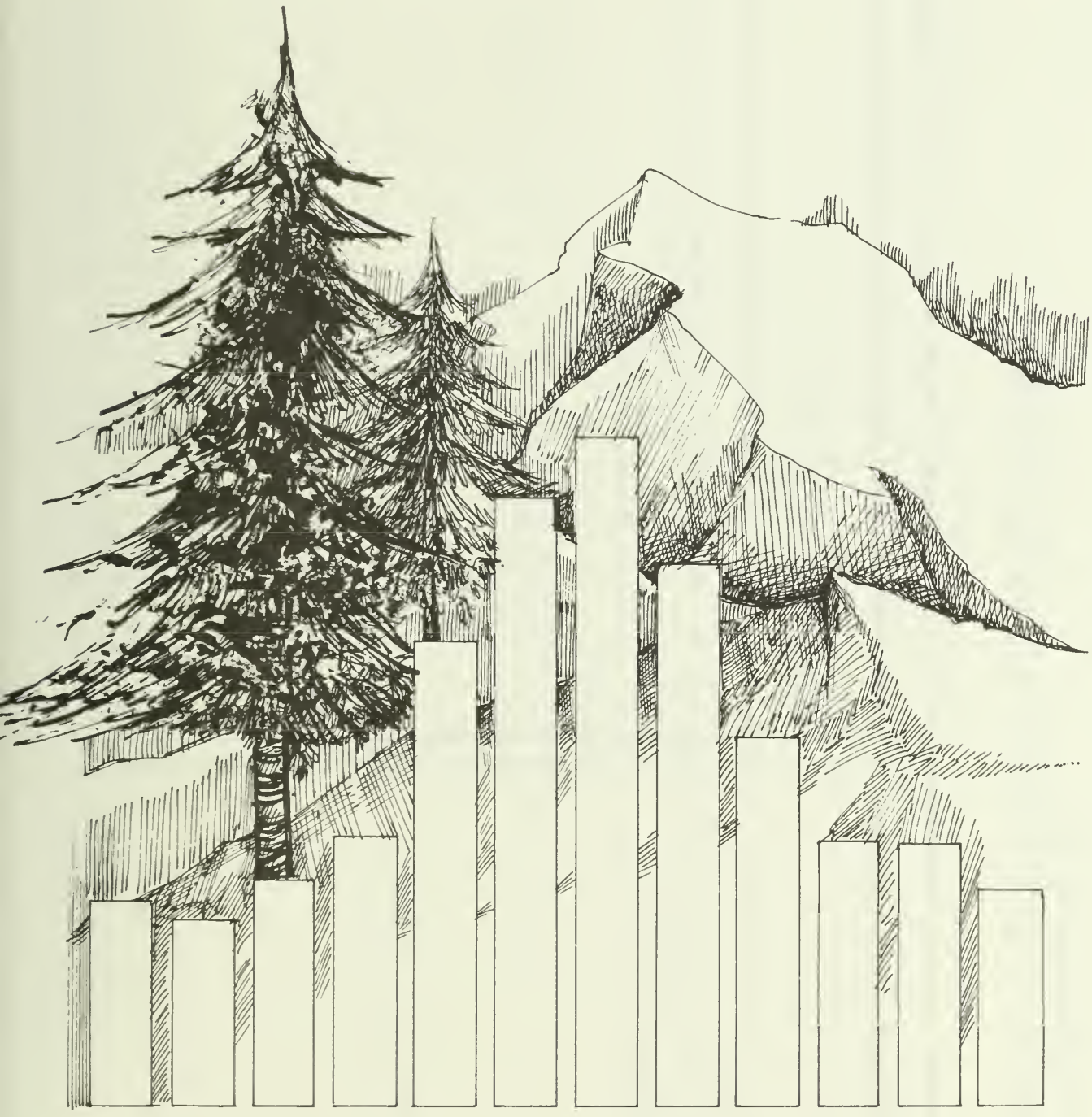
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
4. Solve linear-quadratic and two-quadratic systems.	The four conic sections can be demonstrated using a disectible cone. Then students can be asked to consider the number of possible intersections of a straight line and each of the conic sections. The same can be done for two-quadratic systems. This should demonstrate that, unlike a system of linear equations, we may have as many as four points of intersection. Students are not required to solve these systems by graphical means.		314-317	437,438
H. GEOMETRY				
1. Define and illustrate the following terms related to the circle: radius, chord, interior, exterior, arc, semicircle, segment, sector, central angle, inscribed angle, secant line and tangent line.			205-207	295,296
2.	NOTE: Objectives 2 and 3 are taught only in Math 23.			
3.				
4. Apply the following basic theorems: a) a line containing the centre of a circle that bisects a chord, which is not a diameter, is perpendicular to that chord. (Include corollaries).	Have students draw: 1) The inscribed circle of a triangle. 2) The circumscribed circle of a triangle.	Discuss circular forms in nature, in the community, in art, in industry (e.g., wheels, pulley, gears) and so forth.	207-209	320,321

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
b) a tangent line is perpendicular to the radius drawn to the point of contact. c) The measure of an inscribed angle is half the measure of the central angle subtended by the same arc (or congruent arcs).		Students can produce designs using circles.	209-211 211-213	
5. Construct a tangent to a circle from a given point.			214	
6. Find the length of an arc given the measures of the central angle and the radius.			250-252	41
7. Find the area of a sector given the measures of the central angle and the radius.	For interest, using formulas for perimeter and area, attempt to prove algebraically that the greatest area that can be enclosed within any given perimeter is a circular area.		250-252	41
8. Solve problems involving numerical applications of the above theorems.			208-218 250-252	321
I. TRIGONOMETRY				
1. Maintain previously developed skills in solving right triangles.	This section is intended as a review of the trigonometry studies in Math 10. More complex problems can be introduced here.	a) To find the length of a tunnel through a mountain using an obtuse triangle	230	80-84 392

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES: HOLT MATH 5	PRESCRIBED REFERENCES: MATH IS/5
		 knowing h (height of mountain above AB) and the angles A and B, the length of AB can be calculated. b)  Three holes are to be drilled in a rectangular plate. Find the lengths of a and b, knowing the value of x and one of the acute angles.		
2. Define secant, cosecant and cotangent ratios.			230-232	391
3. Determine the secant, cosecant and cotangent values of acute angles.			241	377,378

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES	
			HOLT MATH 5	MATH IS/5
4. Determine the measure of any acute angle given the secant, cosecant and cotangent ratio.				377,378
5. Solve right triangles using trigonometric ratios.			242,243	391-393
6. Solve problems involving right triangles including angles of elevation and depression.	Students should understand why angles of elevation and depression are equal.		244,245, 246	394-396
7. Determine the relative measures of the sides of: a) a 30 - 60 - 90 triangle b) a 45 - 45 - 90 triangle			228,229	
J. VARIATION				
1. Maintain previous skills of direct, inverse and partial variation.	Partial variation can be extended to include problems involving two equations in two unknowns.		177-180	168-179 183
2. Identify joint variation.			181	180,181
3. Solve problems related to joint variation.			182,183	181,182

MATHEMATICS 23:
PRIMARY REFERENCE CORRELATION



MATHEMATICS 23:
PRIMARY REFERENCE CORRELATION

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
A. RADICALS			
1. Review the basic laws of exponents.	Students should be able to work with monomial bases containing literal and/or numerical factors. Exponents should be restricted to integers, e.g. $(x^3)^{-2}$, $(\frac{2}{3})^3$		117-122
. Identify each part of a radical expression.	Students should be able to identify each part of a radical expression from an example: $\sqrt{x^8}$: Index 2, radicand x^8, radical $\sqrt{x^8}$		
3.	NOTE: Objective 3 is covered only in Math 20.		
4. Simplify radical expressions.	Change pure radicals to mixed radicals and vice versa. Illustrate with radicals of the form: $\sqrt[b]{x^m y^n}$, where b=2 or 3 and x, y $\in I^+$		108-110

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
5. Perform the four basic operations on radicals of the form $\sqrt[b]{x}$, b = 2, 3	Addition and subtraction can be explained by using the distributive laws. For example: a) $2x + 3x = (2 + 3) \cdot x = 5x$ b) $2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$ Restrict the operations with cube roots to simple numerical radicands.	Radical equations can be applied to: a) determining braking distances of a car knowing its speed and type of road. Use: $s = \sqrt{30fd}$, where s = speed f = coefficient of friction and d = braking distance (skid marks) The formula can also be used to find speed by measuring the skid marks and thus determining whether the driver was speeding. b) duration of a hurricane or tornado. Use: $t = 2\sqrt{\frac{d^3}{216}}$ or $t = \frac{d\sqrt{6d}}{18}$ in simplified form where t = time in hours and s = diameter of storm in km.	111-114
6. Rationalize radical denominators that are monomials and binomials.	Demonstrate the reason for dividing by a rational number. It is easier to write: $\frac{\sqrt{10}}{2}$ as a decimal than the unrationalized form of $\frac{\sqrt{5}}{\sqrt{2}}$ which requires division by an approximate decimal. The index should be restricted to 2.		114-117
7. Solve radical equations containing one radical in one variable.	Emphasize the isolation of the radical on one side of the equation before squaring.		
B. POLYNOMIALS			
1. Maintain previous skills in algebraic operations.	This should include both polynomials and rational expressions.		24-32
2. Maintain previous skills of factoring.			

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
3. Maintain previous skills of solving equations with one unknown.	This should include equations containing unsimplified polynomials and some rational expressions.		32-36 42
C. COORDINATE GEOMETRY			
1. Maintain previous skills related to the following: quadrants, axis, origin and ordered pairs.	The usefulness of mathematics increased greatly when mathematicians applied algebraic methods to geometry (in the 17th century) to form a subject called coordinate geometry. Most students are familiar with some aspects of coordinate geometry because they probably had done some work with graphing.	Good references for this section are Math Is 4 and Holt Math 4. Many applications to this section exists in the trades-related fields.	
2. Determine the distance between two points.	Using two points on a graph, say P(1,2) and Q(5,4) derive the distance formula using the Pythagorean Theorem (a review item) before generalizing the formula.	Determine length of rafters, support cables, etc.	66-68
3. Determine the coordinates of the midpoint of a line segment.	By taking two points on a graph, students can predict the midpoint before deriving the midpoint formula.	To determine grade of a road, pitch of a roof, etc.	68-70
4. Determine the slope of the line passing through two given points.	Since coordinate geometry is highly visual students should be encouraged to do neat work so that they could better predict the slope of a line, state relationships between slopes of lines, graph lines from $Ax + By + C = 0$ or $y = mx + b$, and interpret graphs of straight lines.		46-49

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
5. State the relationship between slopes of: a) parallel lines b) perpendicular lines.			56,57
6. Graph lines whose equations are in the form: $Ax + By + C = 0$ or $y = mx + b$ by: a) using ordered pairs b) using the intercepts c) using the slope and y-intercept.			49,50 54,55
7. Interpret graphs of straight lines.			58,60
8. Use the slope test to determine whether three points are collinear.	Example: Determine if P(-4,1), Q(2,3) and R(11,6) are collinear. Slope Test: $m_{PQ} = \frac{1 \quad -3}{-4 \quad -2} = \frac{-2}{6} = \frac{1}{3}$ $m_{QR} = \frac{3 \quad -6}{2 \quad -11} = \frac{-3}{-9} = \frac{1}{3}$ To find the x-intercept the student must understand that along the x-axis, y=0. By putting y=0 in the equation one can get the x-intercept. Similarly, by putting x=0 one can get the y-intercept.		70-72

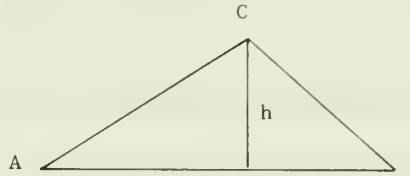
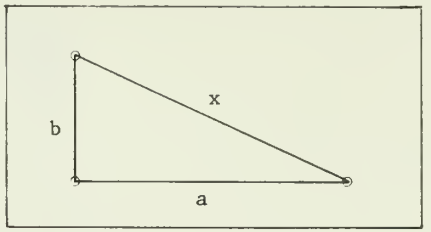
OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
9. State the intercepts of a line by examining the graph.			
10. Determine the intercepts of a line by examining the graph.			
11. Write an equation and draw the graph of: a) a vertical line b) a horizontal line.			
12. Write the equation of a line and draw its graph given a slope and a point on the line.	Again, students should be encouraged to draw reasonable sketches if they are to obtain the required equation or graph.	NOTE: Many good examples of problems related to this section are found in various books. The references to be noted in recent NCTM book, A Sourcebook for Applications of School Mathematics.	50,52
13. Write the equation of a line passing through a given point and a) parallel to a given line b) perpendicular to a given line.			51,52
14. Given two points: a) draw the graph of the line passing through them b) write the equation of the line passing through them.			55,56

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
D. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS			
1. Maintain previous skills.			
2. Draw histograms and cumulative frequency histograms (ogives).	Draw histograms as a review of previous material. (See suggestions in Math 10 applications). Use these histograms to now construct a cumulative frequency histogram. Graphically, (see page 383 of Math Is/4) one can now determine any desired percentile.		
3. Graphically demonstrate how to obtain medians, quartiles and percentiles.			
4. Determine the dispersion of a variable by: a) using percentiles b) calculating the standard deviation.	The dispersion or variability of data is important in any further inference on the source of the data. The standard deviation is the most commonly used measure of dispersion. In many cases, approximately 65% of all data values lie between one standard deviation on either side of the mean. In further statistical work, most tests of inference require a knowledge of the standard deviation. In data with outliers, namely data with a number of small and/or large values, the standard deviation does not accurately represent the dispersion of most of the data. In this case one could perhaps only consider data between the 10th and 90th percentile and use these percentiles as a dispersion measure.	For example, if one wishes to purchase a rifle, there is a considerable range of prices. In this example one pays the most for a rifle with the minimum dispersion, namely the most accurate gun. The dispersion of a rifle can be measured by locking it in a vice and firing a number (say 10) shots into a target. The more scattered the shots appear on the target, the greater the dispersion. As a class project, one could have each student throw ten darts into a dart board. Measure the distances from the bullseye for each dart thrown by a particular student and calculate the standard deviation of these numbers. The student with the smallest standard deviation is the best dart player.	

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
		Another example is the game of roulette. Consider to players, Art and Brunhilda; Art always bets a dollar on either red or black, while Brunhilda always bets a dollar on one of the numbers. After playing all evening, Art and Brunhilda can expect to have lost the same amount of money. However, during the roulette game Art wins approximately half the time, while Burnhilda wins on the average once in every 38 spins of the wheel. Brunhilda's fortune fluctuates wildly while Art's has minimal variability. Since the mean winnings are the same, one can analyze the psychology of a bettor on the basis of the variability of the option he plays.	
E. RELATIONS AND FUNCTIONS	NOTE: Relations and Functions are not taught in Math 23.		
F. QUADRATIC FUNCTIONS, EQUATIONS AND APPLICATIONS	NOTE: Quadratic Functions, Equations and Applications are not taught in Math 23.		

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
G. SYSTEMS OF EQUATIONS			
1. Solve linear systems of equations both algebraically and graphically.	It would be worth introducing solution by graphing before the algebraic solutions are taught. In the graphical solution, a student has a chance to understand how equivalent systems can be generated and used to arrive at a solution.	Problems involving two vehicles travelling at different speeds and leaving a fixed point at different times can be used. Using graphical or algebraic means, we can find out when the two vehicles meet.	79-87
2. Identify systems as having many solutions, one solution or no solution.	Systems can be described graphically showing: a) coincident lines (dependent system) b) parallel lines (inconsistent) c) intersecting lines (independent). Students should be able to determine the type of system by examining the slopes of the lines.		
3. Solve problems based on the solution of systems of equations.	Problems involving statements about numbers, supplementary and complementary angles, speeds of two vehicles, digits of numbers and investments can be used.		88-92

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
H. GEOMETRY			
1. Define and illustrate the following terms related to the circle: radius, chord, interior, exterior, arc, semicircle, segment, sector, central angle, inscribed angle, secant line and tangent line.	Have students draw: 1) the inscribed circle of a triangle 2) the circumscribed circle of a triangle.	Discuss circular forms in nature, in the community, in art and in industry (e.g. wheels, gears, pulleys, etc.). One of the highlights of this section should be the experimental approach to geometry. Through an experimental approach, the students will become sensitized to the real world and many interesting problems will result. For example, why are the tops of all bottles circular? What property of the circle is being taken advantage of?	
2. Discover by experimentation the following circle relationships: a) A line through the centre of a circle and the midpoint of a chord is perpendicular to the chord. (Converses should also be considered). b) The measure of an inscribed angle is half the measure of the central angle subtended by the same arc (or congruent arcs). c) Inscribed angles subtended by the same arc (or congruent arcs) are congruent. d) An angle inscribed in a semi-circle is a right angle. e) A tangent line is perpendicular to the radius drawn to the point of contact. f) Tangent segments drawn to a circle from the same exterior points are congruent.			

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
3. Solve problems involving numerical applications of the relationships in Objective 2.			
I. TRIGONOMETRY			
1. Maintain previously developed skills in solving right triangles.	This section is intended as a review of the trigonometry studied in Grade 10 Math. More complex problems can be introduced here.	a) To find the length of a tunnel through a mountain using an obtuse triangle  knowing h (height of mountain above AB) and the angles A and B, the length of AB can be calculated. b)  Three holes are to be drilled in a rectangular plate. Find the lengths of a and b, knowing the value of x and one of the acute angles.	163-170

OBJECTIVES	COMMENTS	APPLICATIONS	PRESCRIBED REFERENCES AMT
2. Define secant, cosecant and cotangent ratios.			158,159
3. Determine the secant, cosecant and cotangent values of acute angles.			165-167
4. Determine the measure of any acute angle given the secant, cosecant and cotangent ratio.			158,159
5. Solve right triangles using trigonometric ratios.			170-173
6. Solve problems involving right triangles including angles of elevation and depression.	Students should understand why angles of elevation and depression are equal.		173-193
7. Determine the relative measures of the sides of: a) a 30 - 60 - 90 triangle b) a 45 - 45 - 90 triangle			159-161
J. VARIATION			
1. Maintain previous skills of direct, inverse and partial variation.			18-21
2. Identify joint variation.			
3. Solve problems related to joint variation.			

MATHEMATICS ELECTIVES

ABSOLUTE VALUE

RECOMMENDATIONS: MATH 20, 30. 23,33

In some problems only the magnitude of a value is required. E.g., How much did the temperature increase from 9:00 a.m. to 3:00 p.m. if it were - 8°C at 9:00 a.m. and 4°C at 3:00 p.m.?

We would disregard the direction on a number line in order to solve the above. In other words, we would use the absolute value of the numbers.

References:

1. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1978.

3. Carter, J., Clark, J., Porter, B., and Stouffer, M., *Mathematics Alive 3*, Toronto: Copp Clark Publishing, 1978.

4. Bye, M., and Elliott, H., *Math Probe 3*, Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Use the absolute value symbol correctly.	
2. Use absolute value to indicate the measure of the distance between the point and the origin on a number line.	
3. Use absolute value to tell the distance between any two points on the number line.	
4. Solve equations like $\lvert 2x - 4 \rvert = 8$	
5. Find the solution sets of inequalities like $\lvert 3x \rvert < 12$ and graph the solution set.	

AREA AND VOLUME

RECOMMENDATIONS: MATH 20, 30, 23, 33

Observation tells us that geometric figures are common in everyday life. It is often necessary to find areas or volumes in order to solve everyday problems.

References:

1. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Applied Mathematics for Today: Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1980.

2. Ebos, F., and Tuck, B., *Math is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1970.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Determine the area (using the pertinent formula) for each of the following plane figures: triangles, squares, rectangles, parallelograms, trapezoids and circles.	Using different floor plans have students find the area of floors so that wall-to-wall carpet may be laid.
2. Determine the total area (or area of different regions) of various polyhedrons (prisms and pyramids), cylinders and cones.	Wall papering is popular. Have students find the quantity of wallpaper needed to cover walls in rooms of various shapes. Wallpaper (or adhesive covering material) is often used to decorate various household items like wastepaper baskets. Have students suggest different items and calculate the quantity of covering required.
3. Determine the volume of various polyhedrons (prisms and pyramids), cylinders, cones and spheres.	Using structural forms with various dimensions students could calculate the number of M^3 of concrete to be used. The capacity of swimming pools, silos, etc., of varying shapes and dimensions could be calculated.

NOTE: A review of (a) The theorem of Pythagoras and (b) π might be required.

ARRANGEMENTS AND SELECTIONS

RECOMMENDATIONS:
MATH 30, 33

References:

1. Nichols, E., Heimer, R., and Garland, E., *Modern Intermediate Algebra*, Toronto: Holt, Rinehart and Winston of Canada.

2. Johnson, R., Lendsey, L., Slesnick, W., and Bates, G., *Algebra and Trigonometry*, Don Mills, Ontario: Addison-Wesley Publishing Company.

3. Elliott, H., Fryer, K., and Gardner, J., *Algebraic Structures and Probability*, Toronto: Holt, Rinehart and Winston of Canada.

4. Travers, Dalton et al, *Using Advanced Algebra*, Toronto: Doubleday Canada Ltd.

5. Dolciani, M., Berman, S., and Wooton, W., *Modern Algebra and Trigonometry*, Don Mills: Thomas Nelson and Sons (Canada) Ltd.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Evaluate and/or simplify expressions involving factorials and/or ${}^{n}P_{r}$.	
2. Solve word problems involving linear arrangements of n different objects.	
3. Solve word problems involving a linear arrangement of r objects, given n different objects.	
4. Solve word problems involving linear arrangements of one or more objects at a time, given n different objects.	

OBJECTIVES	COMMENTS/ ACTIVITIES
5. Solve word problems involving linear arrangements of n objects, if some of the objects are the same.	
6. Solve word problems involving a restricted linear arrangement of objects.	
7. Solve equations involving the application of the formula $${}_nP_r = \frac{n!}{(n-r)!}$$	
8. Solve word problems involving the application of circular arrangements of n different objects.	
9. Solve word problems involving circular arrangements of n different objects, with restrictions.	
10. Solve word problems involving circular arrangements on a ring.	
11. Evaluate and/or simplify expressions involving factorials and/or ${}_nC_r$.	
12. Solve word problems involving a selection of r objects, from n different objects.	
13. Solve word problems involving selections of one or more objects at a time, from n different objects.	

OBJECTIVES	COMMENTS/ ACTIVITIES
14. Solve word problems involving a restricted selection of objects.	
15. Solve equations involving the application of the formula $_nC_r = \frac{n!}{r!\ (n-r)!}$	

COMPLEX NUMBERS

RECOMMENDATIONS: MATH 20,30,33

Complex numbers are a natural outgrowth of the study of quadratic equations. Development of a new number system through the introduction of a new symbol can be illustrated. For example, in expanding the system No to I, we merely introduce a new symbol: the minus sign. Similarly for the complex numbers, we introduce a new symbol, i, to represent $\sqrt{-1}$. This allows us to solve quadratic equations which have no solution in the real number system.

Although few practical applications of complex numbers exist at a level which high school students can understand, complex members are used extensively by electrical engineers. Several examples of this, at a very basic level, are included in reference 3.

References:

1. *Algebra Two and Trigonometry*, Vogeli et al, Silver-Burdett Company.

2. *Holt Algebra 2 With Trigonometry*, Nichols et al, Holt, Rinehart and Winston, 1974.

3. *Using Advanced Algebra*, Travers et al, Laidlaw Publishers, 1975.

4. *Algebra and Trigonometry: Structure and Method (Book 2)*, Dolciani et al, Houghton, Mifflin, 1977.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Demonstrate the need for a new number system beyond the real numbers in order to solve equations of the form $x^2 + 1 = 0$.	
2. (a) Perform simple arithmetic operations with complex numbers of the form a + bi. (b) Evaluate the congugate and use it in division of complex numbers. (c) Solve for unknowns for equations in the form 2 + 3i = x + yi.	

OBJECTIVES	COMMENTS/ ACTIVITIES
3. Graph complex numbers on an Argand diagram, (rectangular, coordinate system) using a vector to represent the complex number.	
4. Evaluate the absolute value (modules) of a complex number and interpret it as the length of a vector on an Argand diagram.	
5. Solve quadratic equations which have non-real roots.	
6. Solve higher degree equations by special factoring methods (sum or differences of cubes, differences of squares).	
7. Find roots of a number algebraically. (e.g., $x^3 + 1 = 0$ can be used to find the 3 cube roots of -1).	
8. Find the roots of a number of graphical means, using an Argand diagram. (All roots represented by vectors with an equal length).	
9. Solve application problems of complex numbers (impedence of an electrical circuit).	

COMPUTER LITERACY

RECOMMENDATIONS: MATH 20, 30, 23, 33

Many of today's students are becoming acquainted with the workings of the computer and the many uses to which it can be put. An elective component of computer literacy should provide the student with an introduction to the field of computers with a provision for students who are interested to continue work in computer science.

References:

1. *The Mathematics Teacher,* National Council of Teachers of Mathematics, February, 1980

2. Brown, J. R., *Instant Basics.* Dilitherim Press, Forest Grove, Oregon, 1978.

3. Dwyer, T. and Critchfield M., *Basis and the Personal Computer,* Addison-Wesley, Don Mills, Ontario, 1978.

4. Lien, David, *Radio Shack Computer Language.* Fort Worth, Texas: Tandy Corporation, 1978.

At present there are a variety of computer companies with many changes occuring rapidly. It would be adviseable to become acquainted with the variety of hardware and software available on the market.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Develop an appreciation of the role of computers in our society.	Give examples of how computers are used in data processing, mathematics, science and other fields. Describe the general categories and types of computers in society. Summarize the historical events and more recent developments that have resulted in present-day computers. Describe the duties and education needed for various types of computer-related occupations. Give examples of how computers have been misused in society.

OBJECTIVES	COMMENTS/ACTIVITIES
	Define concepts like artificial intelligence, dehumanization, miniaturization.
2. Obtain basic understanding of the computer as a machine: its capabilities and limitations.	Identify the essential components of a modern computer. Identify the function and relation of each component to the whole computing system. State whether or not a given problem is suitable for solution on a computer. Give examples of problems or situations that cannot be solved by a computer.
3. Develop the algorithmic process of thought.	Solve standard types of mathematical and logical problems by using various techniques and strategies. Give an example of an algorithm. Flowchart a simple algorithm.
4. Acquire a working knowledge of a programming language.	Differentiate between machine language, a programming language and job control language. Prepare and run a simple computer program using the fundamental features of the language BASIC. Program in BASIC using advanced features like looping, subscripted variables, character strings and library functions.
5. Develop skills enabling the student to use the computer for probelm solving.	Solve a given mathematics or data processing problem by using a computer. Describe the solution to a problem by providing the appropriate documentation, flowcharts, program listing and sample runs.

CONSUMER MATHEMATICS

RECOMMENDATIONS: Math 20, 30, 23, 33

It is becoming more and more important that students have some understanding of how mathematics is related to the consumer. A working knowledge of how the basic skills in mathematics are applied to consumer topics such as accessing money, spending money and the management of money is essential for a student to become a good citizen.

References:

1. *Mathematics in Life,* Gage Publishing Co.
2. *Mathematics Plus,* Houghton Mifflin Publishing Co.
3. *Mathematics for the Real World,* Merrill/Bell and Howell Publishing Co.
4. *Mathematics for Daily Use,* Doubleday Publishing Co.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. To develop an appreciation for how money is obtained. (a) Jobs - Descriptions - Applications - Wages	Identify different careers and discuss the wages paid for the related work. Identify what is needed for the accomplishement of different jobs. Discuss filling out applications and interviews.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. (Continued)	
(b) Payroll - Methods of payment - Deduction · Tax · Pension · Benefits	Have students discuss and analyze real or imaginary paychecks.
(c) Selling Products	Have students participate in such activities as a school store, concession or canteen.
(d) Record Keeping	Discuss and practice the necessity for good record keeping.
2. To develop an understanding of wise and thrifty spending habits.	
(a) Buying - Comparative buying	Students can check out the cost of a shopping list of some everyday needs. Prices can be compared from store to store, by brand and by size. Prices could be prepared over a period of time to determine price changes and trends.
(b) Credit Buying - Types of credit - Types of payment - Interest rates - Installment buying - Sources of borrowed money	Students could determine the cost of buying such items as a car.
(c) Operation Costs	Students could determine the cost of operating a car for a period of a year. Things to take into account are: purchase price, licence, insurance, gasoline, service and maintenance, depreciation.

OBJECTIVES	COMMENTS/ ACTIVITIES
2. (Continued)	
(d) Bank Accounts	Students can study the various bank accounts probably best by an organized field trip to a local bank.
(e) Food, Shelter and Clothing	Generate activities with the students to have them role-play situations about the basic necessities of life.
3. Mathematics of Money Management	
(a) Insurance - Life - Property - Automobile	Have students study the various types of insurance programs in order to determine the advantages and disadvantages of each type.
(b) Home Ownership and Renting	Have students discuss the sale and rental of homes with the local real estate firms.
(c) Taxation	Make a study of income tax and fill out an income tax form. Discuss the local tax structures within a community.
(d) Investments	Have students make a study of the stock market.

HISTORY OF MATHEMATICS

RECOMMENDATIONS: MATH 20, 30, 23, 33

The history of mathematics can provide many interesting discussions and lends itself to many interesting projects for the classroom. A look at historical topics is unlimited and is dependent only upon the creativity of the teacher.

Reference:

1. *Historical Topics for the Mathematics Classroom*, Thirty-first yearbook, National Council of Teachers of Mathematics, 1969.

NOTE: This yearbook also contains an excellent bibliography of the many books and articles available on historical topics in Mathematics.

OBJECTIVES	COMMENTS / ACTIVITIES
1. To acquire an appreciation for the historical development of mathematics from counting to modern day computers.	Discuss various types of early developments in mathematics. Many of these can be set in activity stations, as display areas or in written project form.
2. To humanize mathematics by looking at the lives of some of the mathematicians.	Investigate the lives of the famous mathematicians by using different methods such as: reporting, writing essays, role-playing, discussions, films.
3. To interrelate mathematics with other subject areas such as science, music, social studies and art.	
4. To familiarize students with the use of various mathematical instruments which have been developed throughout the year.	Discuss, demonstrate the use of, or construct the basic mathematical instruments for:

OBJECTIVES	COMMENTS / ACTIVITIES
4. (Continued)	(a) Measurement - sundial - waterclock - transit - sextant - angle mirror - micrometer - caliper - trundle wheel (b) Calculations - slide rule - logarithms - calculators - napier's bones - abacus - computers

INEQUALITIES

RECOMMENDATIONS: MATH 20, 30, 23, 33

All or most of our past experience in mathematics has dealt with equalities. We can make use of this experience to solve inequalities since inequalities are solved in the same manner as equalities except when multiplying or dividing by a negative. In this case the direction of the inequality sign must be changed.

References:

1. Burns, A., Pinkney, R., and Del Grande, J. *Mathematics for a Modern World: Book 3;* Second Edition, Toronto, Gage Educational Publishing Ltd., 1976.

2. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

OBJECTIVES	COMMENTS /ACTIVITIES
1. Solve inequalities.	
2. Graph the solution sets of inequalities.	

LINEAR PROGRAMMING

RECOMMENDATIONS: MATH 20, 30, 33

Linear programming involves the attempt to maximize results and minimize efforts to produce those results. Such attempts originated during World War II when the Allies attempted to maximize production and minimize costs. Manufacturing problems involving numbers of items and various other constraints, such as time required for production, can be analyzed in a systematic way using linear equations and inequalities.

The fundamental assumption of linear programming is: the maximum value of the parameter P, for the relation $P = Ax + By$, occurs at one of the vertices of the polygonal region determined by the various constraints in the problem. The maximum value of P can be found by examining these vertices.

References:

1. Ebos and Tuck, *Math Is 4*, Thomas Nelson and Sons, 1979.

2. Travers, Dalton et al, *Using Advanced Algebra*, Doubleday, 1977.

3. Wigle, Jenning and Dowling, *Mathematical Pursuits Three*, Macmillan of Canada, 1977.

4. Dotton, Knill and Seymour, *Applied Mathematics for Today*, McGraw-Hill Ryerson, 1976.

5. Hanwell, Bye and Griffiths, *Holt Mathematics 4* (Second Edition), Holt, Rinehart and Winston, 1980.

OBJECTIVES	COMMENTS / ACTIVITIES
1. Define the following terms: parameter, region, constraint, maximum point.	
2. Draw the graph of a linear equation in the form $Ax + By = C$.	
3. Draw the graph of a region defined by an inequality.	

OBJECTIVES	COMMENTS/ ACTIVITIES
4. Determine a region bounded by several inequalities.	
5. Find the intersection of 2 linear equations by graphical or algebraic means.	
6. Find the maximum value of a perimeter P defined by $P = Ax + By$ for a given set of constraints.	
7. Determine the value of parameter P at each of the vertices of the polygon which results from graphing all the constraint conditions on x and y.	
8. Apply linear programming to solving business-oriented problems.	

MATH ART

RECOMMENDATIONS: MATH 20, 30, 23, 33

This unit is intended to give students an opportunity to apply in an interesting way some of the skills and concepts learned in Geometry (both Euclidean and Analytic). Once a student learns some of the simple principles of the artwork, he can create his own designs.

References:

1. *Art 'N Math*, Billings, Campbell and Schwandt (Action Math Associates, Inc.).

2. *Creating Escher - Type Drawings*, Rannucci and Teeters (Creative Publications).

3. *Graph Gallery*, Boyle (Creative Publications).

4. *Paper Folding in the Classroom*, Johnson (NCTM Publications).

5. *Creative Constructions*, Seymour (Creative Publications).

6. *Line Designs*, Seymour (Creative Publications).

7. *Mathematics Teacher (NCTM)*.

   (a) Tangram Mathematics, February, 1977, pp. 143-146.
   (b) The Artist as Mathematician, April, 1977, pp. 298-308.
   (c) Filing (tesselations), March, 1978, pp. 199-202.
   (d) Transformation Geometry and the Artwork of M.C. Escher, December, 1976, pp. 647-652.
   (e) Creativity With Colors, March, 1976, pp. 215-218.

8. *Geometry*, Jacobs (W.H. Freeman Publishing).

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Graphing Pictures (a) Given a set of ordered pairs, locate these on a coordinate system and join them to form a picture.	

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Graphing Pictures (continued) (b) Given a set of linear equations, each with a restricted domain, draw the lines on a coordinate system to form a picture.	Student writes his name or suitable message in block letters on graph paper, then analyzes each letter to write the equations which represent them (restrict to straight lines).
2. Line Designs (a) Draw line designs within one angle.	Student creates his own pattern using basic construction techniques for angles. Alternate rectangles can be colored to form a pattern. The same approach can be used with colored thread and small nails on a sheet of plywood to create string designs.
3. Geometric Constructions (a) Perform basic constructions related to line segments, angles, perpendicular and parallel lines. (b) Draw a polygon within a circle using ruler and compass (e.g., hexagon, polygon, square).	Student analyzes and recreates simple drawings from patterns. Student creates and colors his own designs.
4. Tesselations (a) Draw a tesselation based on a rectangle, a triangle or other polygons.	Students find examples of tesselations; e.g., linoleum, floor tiles, wallpaper, brick wall, etc.
5. Escher Drawings (a) (An extension of tesselations). Draw a simple tesselation based on Escher's work and analyze how it is drawn.	Students create their own Escher - type drawings. Students analyze some of Escher's paintings to find geometric contradictions (e.g., "Belvoir").

OBJECTIVES	COMMENTS/ ACTIVITIES
6. Paper Folding (a) Demonstrate the following concepts using paper folding: - angle bisector - bisection of a segment - line perpendicular to a given line - centre of a circle (b) Construct a parabola, hyperbola and ellipse using paper folding.	
7. Tangrams (a) Rearrange polygonal shapes to form geometric figures. (b) Illustrate basic properties of polygons (e.g., form an isosceles trapezoid and show the non-parallel sides are equal by moving the component parts around).	Using the 7 pieces of a tangram square, form rectangles, isosceles triangles, parallelograms and trapezoids. Use the tangram pieces to make up figures such as the letter "T", the number "1", a house, a cat, etc.

MATHEMATICAL INDUCTION

RECOMMENDATION: MATH 30

The name, mathematical induction, is somewhat misleading because the process is deductive in nature, leading to a firm conclusion. It is usually employed in proving the validity of a statement involving all positive integral values of "n".

For example, one method of finding a square root on an ordinary computing or adding machine is based on the fact that the sum of "n" odd integers is equal to n^2.

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + - - - + (2n-1) = n^2$$

If this can be proven for (a) specific values of n and, (b) when nth position = k as well as k + 1 where right side equals left side then the conclusion (c) is verified.

References:

1. Johnson, R.E., Lendsey, L.L., Slesnich, W.E., Bates, G.E., *Algebra and Trigonometry*, London: Addison-Wesley Publishing Co., 1976.

2. Del Grande, J.J., Duff, G.F.D., Egsgard. J.C., *Mathematics 12 - Third Edition*, Toronto: Gage Publishing Ltd., 1980.

3. Vance, E.P., *Mathematical Induction and Conic Sections*, London: Addison-Wesley Publishing Co., 1971.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use mathematical induction to prove various additive algorithms.	

OBJECTIVES	COMMENTS / ACTIVITIES
2. Use mathematical induction to prove various division algorithms.	
3. Use mathematical induction to prove trigonometric algorithms.	

MATHEMATICS FOR INDUSTRY

RECOMMENDATIONS: MATH 20, 30, 23, 33

Many practical applications for mathematics are available through the Industrial Arts department. Consultation with the Industrial Arts teachers can provide many interesting projects for students.

For example, the Math Modules produced by General Publishing Company provide many examples in the areas of drafting, machine shops and consumer mathematics.

Preparation of units, projects and activities in this section are completely dependent upon the creativity of the teacher.

MATHEMATICS OF ACCOUNTING

RECOMMENDATIONS: MATH 20
30, 23, 33

The value of accounting skills in the everyday life of all people has increased dramatically in recent years. As a result all students could benefit from a basic knowledge of how mathematics is applied to the field of accounting. The treatment of basic accounting procedures could be designed to permit interested students to further pursue accounting courses in the Business area.

References:

The Business Education deparment of the school should have many practical references to use. Consultation with the Business Education teachers would be desirable in setting up an elective in this area.

OBJECTIVES	COMMENTS / ACTIVITIES
1. Emphasize the importance of maintaining adequate accounting records in business and personal life.	
2. Emphasize the importance of basic mathematics skills to the field of accounting.	
3. Improve ability in problem-solving and logical thinking through real-life applications.	
4. Develop traits of neatness, accuracy and the ability to interpret and analyze basic accounting records.	

MATRICES

RECOMMENDATIONS: MATH 20, 30,33

Tables are very useful ways of arranging information. Data from tables can be set in a more concise form called a matrix where we write this data in columns and rows.

Reference:

1. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

OBJECTIVES	COMMENTS / ACTIVITIES
1. To recognize and use the matrix as a concise form of arranging information.	Construct a 3 x 3 magic square. Then rearrange the numbers in the cells so that the sum of each new row, column and diagonal is also equal. How many arrangements are there if the square is to remain a magic square? Do likewise for a 4 x 4 and a 5 x 5 magic square.
2. To comprehend the difference between columns and rows.	Using everyday situations construct matrices with different orders. E.g., team records.
3. To multiply matrices.	
4. To apply matrices in solving problems.	

POLYNOMIAL FUNCTIONS

RECOMMENDATIONS: MATH 30

The study of polynomial functions is an opportunity to extend the previously studied topics of polynomial functions, linear and quadratic. Such algebraic processes as factoring and division become steps in a process to determine the roots of polynomial equations of second degree or higher. Sketch the graphs of polynomial functions.

References:

1. Nichols, El, Heimer, R., and Garland, E., *Modern Intermediate Algebra,* Toronto: Holt, Rinehart and Winston, 1970.

2. Del Grande, J., Duff, G., and Egsgard, J., *Mathematics 12 (Third Edition),* Toronto: Gage Publishing Ltd., 1980.

3. Dolciani, M., Berman, S., and Wooton, W., *Modern Algebra and Trigonometry,* Don Mills: Thomas Nelson and Sons (Canada) Ltd., 1964.

4. Travers, K., Dalton, L., and Brunner, V., *Using Advanced Algebra,* Toronto: Doubleday Canada Ltd., 1976.

OBJECTIVES	COMMENTS / ACTIVITIES
1. Recognize a polynomial function as a function of the form $f(x) = a_o x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_{n-1} x + a_n,\ a_o \neq 0,$ $n \in N.$	
2. Classify a given polynomial function according to (a) degree, and (b) coefficients (integral, rational, or real).	
3. Evaluate an integral polynomial function for a given value of the domain by substitution.	

OBJECTIVES	COMMENTS / ACTIVITIES
4. Divide an integral polynomial in one variable by a binomial of the form (x - a) using the division algorithm.	
5. Divide an integral polynomial in one variable by a binomial of the form (x - a) using synthetic division.	
6. Evaluate an integral polynomial function for a given value of the domain using the Remainder Theorem.	
7. Determine whether a given value of the domain is a zero of a given integral polynomial function.	
8. Determine potential integral zeros of a given integral polynomial function.	
9. Factor a given integral polynomial over the integers.	
10. Determine potential rational zeros of a given rational polynomial function.	
11. Determine the integral zeros of a given integral polynomial function.	
12. Determine the rational zeros of a given rational polynomial function.	

OBJECTIVES	COMMENTS/ACTIVITIES
13. Determine an integral polynomial function given rational zeros.	
14. Determine the possible sums of multiplicities of zeros of a given real polynomial function.	
15. Determine all zeros of a given real polynomial function.	
16. Sketch the graph of a given polynomial function.	

PROBABILITY

RECOMMENDATIONS: MATH 30, 33

There are numerous books which cover this topic, usually together with some statistics. In a short elective component, it is important to just get the feel of calculating some interesting probabilities. If one can experimentally verify or disprove one' results, so much the better.

OBJECTIVES	COMMENTS/ACTIVITIES
The following concepts should be taughts as a preamble: 1. The idea of an event E as the outcome of some experiment of trial.	If the above five ideas can be mastered, the one can follow with some simple combinatorial results before some interesting examples can be tried. The student should learn about: a) Factorial notation
2. The meaning of $P(E)$, the probability of this event occurring. Also the fact that probabilities are number between 0 and 1.	b) that $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$ represents the number of ways of arranging k objects among n locations.
3. The concept of independence of events.	c) the hypergeometric distribution.
4. The meaning of mutually exclusive events.	One has a box containing N_1 black marbles and N_2 white marbles for a total of $N = N_1 + N_2$ marbles. If one now n marbles from the box (at random without replacement) then the probability one obtains n_1 black and n_2 white marbles is $\binom{N_1}{n_1}\binom{n_2}{n_2}\Big/\binom{N}{n}$
5. If one has two events A and B, then: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.	e.g., if the box contains 5 black and 3 white marbles, then in drawing 2 marbles the probability one obtains one of each colour is: $\binom{5}{1}\binom{3}{1}\Big/\binom{8}{2} = {}^{15}/_{28}$

OBJECTIVES	COMMENTS/ACTIVITIES
continued	With these minimal tools, one can now tackle many interesting problems. For example, when dealt a 5-card hand from a standard deck, following are the poker hands one can obtain: a) nothing b) one pair c) 2 pairs d) 3 of a kind (a triple) e) full house (one triple, one pair) f) 4 of a kind g) flush (all in one suit) h) straight (5 cards in sequence) i) straight flush (5 cards in sequence in one suit) j) royal flush (highest straight flush)

If one considers these 10 events as mutually exclusive, one can calculate the probability of each occurring and hence order, via probabilities, the sequence of winning events.

There are a multitude of variations on this one theme alone, namely what happens if you have a wild card? Does this change the order of winning events? All these can be answered, and if in doubt, deal a few hands to check your answer.

Reference:

1. Most of these notations can be found in any respectable text. One in which the card hands are discussed and the probabilities calculated is: *Basic Probability and Applications*, by M. Nosal., Published by W. B. Saunders.

TOPOLOGY

RECOMMENDATIONS: MATH 20, 30, 23, 33

Reference:

1. *Excursions into Mathematics*, by Beck, Bleicher and Crowe. Published by Worth.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. Eulers formula.	Pages 3 - 11.
2. The number of regular polyhedra.	Pages 12 - 16.
3. Tesselation of the plane: Use Eulers formula to show that triangles, hexagons and squares are the only regular polygons one can use to tile a floor.	
4. A European "football" is made of pieces of leather in the shape of regular pentagons and regular hexagons. These are sewed together so that each pentagon is surrounded by hexagons and each hexagon is surrounded (alternately) by three pentagons and three hexagons. Determine the number of pentagons and hexagons of such a football.	
5. Deltahedra: this section deals with non-regular polyhedra, all of whose faces are triangles. It also illustrates methods of constructing these polyhedra out of cardboard.	NOTE: The objectives listed are all fully illustrated in the text with numerous examples that can actually be constructed.
6. If time permits, there is a section of polyhedra without diagonals using Euler's formula.	

TRANSFORMATIONAL GEOMETRY

RECOMMENDATIONS: MATH 20
30, 23, 33

Euclid has been criticized for his use of the super-position argument to prove the familiar side-angle-side (SAS) congruence theorem. Simplified, the argument calls for one to pick up a triangle and attempt to fit it on another. While deficiences have sometimes been pointed out in Euclid's system of axioms, it may be noted that those seeking to evolve more logically satisfactory systems of axioms have generally attempted to by-pass the difficulty. Although it may be necessary to justify every step of a proof one should not lose the intuitive appeal of Euclid's approach. Most geometry teachers proably explain congruence by placing one triangle over another. E.g., by cutting out cardboard triangles, by tracing one triangle over another, etc.

Basically, then, we could consider the result of mapping a figure by some appropriate transformation.

References:

1. Bye, M., Griffiths, T. and Hanwell, A., *Holt Math 4*, Toronto: Holt, Rinehart and Winston, 1980, pages 265-303.

2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: An Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1977, pages 320-344.

3. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979, pages 353-371.

4. *The Mathematics Teacher (NCTM): Transformation Geometry and the Artwork of M.C. Escher*, December 1976, pages 647-652.

OBJECTIVES	COMMENTS/ ACTIVITIES
1. To develop and describe the following different types of transformations: a) Translations b) Reflections c) Rotations d) Dilations	Graph paper should be used whenever possible. (For obvious reasons, these are also referred to as slides or glides.) A transparent plastic reflector called a MIRA would be an excellent tool in teaching reflections. The hands of a clock noting its centre is a very useful example. Most students know what similarity (same shape, different size) means. Use examples (photo enlarger, microscope, etc.), to explain dilations.

TRIGONOMETRIC IDENTITIES

RECOMMENDATION: MATH 30

Trigonometric equations which are true for all permissible values are called trigonometric identities. There are eight fundamental trigonometric identities:

1. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
2. $\cot \theta = \frac{\cos \theta}{\sin \theta}$
3. $\csc \theta = \frac{1}{\sin \theta}$
4. $\sec \theta = \frac{1}{\cos \theta}$
5. $\tan \theta = \frac{1}{\cot \theta}$
6. $\sin^2 \theta + \cos^2 \theta = 1$
7. $\tan^2 \theta + 1 = \sec^2 \theta$
8. $1 + \cot^2 \theta = \csc^2 \theta$

These fundamental identities can be used to develop or prove more complex trigonometric identities. To prove a trigonometric identity one should adopt the following objectives.

References:

1. Travers, K.J., Dalton, L.C., Brunner, V.F., *Using Advanced Algebra,* Toronto: Doubleday Canada Ltd., 1975.
2. Johnson, R.E., Lendsey, L.L., Slesnick, W.E., Bates, G.E., *Algebra and Trigonometry,* London: Addison-Wesley Publishing Co., 1976.
3. Dottori, D., Knill, G., Stewart, J., *Foundations of Mathematics for Tomorrow - Senior SE Metric Edition,* Toronto: McGraw-Hill Ryerson Ltd., 1978.
4. Del Grande, J.J., Duff, G.F.D., Egsgard, J.C., *Mathematics 12 - Third Edition,* Toronto: Gage Publishing Ltd., 1980.
5. Welchons, A.M., Krickenberger, W.R., *Trigonometry With Tables,* Toronto: Ginn and Company, 1957.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use fundamental identities to develop or prove more complex trigonometric identities.	Work with each side separately. Substitute fundamental or proven identities into both sides. Do any subtraction, addition, multiplication, division or simplification that may be necessary. Try to get each side into identical format and hence prove the identity.

VECTORS

RECOMMENDATIONS: MATH 20, 30,33

Vectors provide a useful tool for analyzing problems that deal with trips, forces, velocities, accelerations and displacement. A vector is a quantity that requires both a magnitude and direction to describe it.

References:

1. Bye, M. and Elliott, H., *Math Probe 3*, Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.

2. Burns, A.G. Pinkney, R.G., and Del Grande, J.J., *Mathematics for a Modern World: Book 3*, Toronto, Gage Educational Publishing Ltd., 1976.

3. Dottori, D., Knill, G. and Seymour, J., *Applied Mathematics for Today: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1976.

4. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1974. Nichole, *Modern Intermediate Algebra*.

5. Dottori, D., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1978.

6. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

7. Hilton, A., *Vectors*, London: Macdonald Educational Colour Units, 1976.

OBJECTIVES	COMMENTS / ACTIVITIES
1. Understand those phenomena which possess magnitude and direction.	Describe five situations in which vectors are used.
2. Add and substract vectors.	Create and solve problems using vectors.
3. Multiply a vector by a scalar.	
4. Solve problems using vectors.	

APPENDIX A

ADDITIONAL COMMENTS

Alberta Education would welcome any comments and suggestions pertaining to the Grade 20/23 Interim Curriculum Guide. Please contact or forward comments to the Regional Office Mathematics Consultant in your area.

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